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Thermoacoustic Oscillations in Combustion Chambers of Gas Turbines

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This paper presents an overview of thermoacoustic oscillations in modern gas turbine combustors with premixed combustion. In conventional combustors a substantial percentage of air enters downstream of the primary zone of combustion. As a consequence, the liners of conventional combustors are very powerful sound attenuators. In a modern premixing combustor effectively all of the air enters through the burners, and there is almost no sound attenuation downstream of the primary zone of combustion. The combination of this problem with the difficulties to stabilize premixed flames has led to a situation where thermoacoustic stability has become the key issue of modern combustion technology. The relative importance of a variety of self-excited oscillations and oscillations that are forced by aerodynamic instabilities is investigated. It is found that the two leading excitation mechanisms are associated with periodic (lean) extinction and with vortex rollup in the primary zone of combustion. For the latter case a simple combustor model is used to study the amplification effects due to resonant wave motion and coincidence of mechanical eigenfrequencies of combustor walls with excitation frequencies.

Nomenclature

A, B, C	= constants related to physical properties
A_0	= amplitude of velocity disturbance
a	= sound speed
a_U	= sound speed upstream of contraction
c_p	= specific heat at constant pressure
f	= Riemann invariant of downstream running wave
g	= Riemann invariant of upstream running wave
L	= combustor length
l	= length of plenum chamber
M	= Mach number
M_U, M_D	= Mach numbers upstream and downstream of contraction
n	= integer, mode number
p	= pressure
Q	= rate of heat addition per unit area
$\hat{s}$	= amplitude of entropy disturbance
T	= temperature
t	= time
u	= flow speed
u_F	= relative flame speed with respect to approaching fresh mixture
u_B	= relative flame speed with respect to burnt gases on the downstream side
X	= axial position of flame averaged over cross-sectional area of combustor
x	= axial coordinate
α	= effective constant of elasticity per unit area of elastic wall
γ	= ratio of specific heats
Δ	= small-amplitude disturbance of a quantity if placed in front of the symbol for the corresponding quantity
ΔQ^*	= fluctuation in rate of heat addition that would occur in the absence of fluctuations in effective flame position
ΔQ^{**}	= contribution to fluctuation in rate of heat addition due to fluctuations in effective flame position caused by flow instabilities, for example
Δu^*	= velocity fluctuation that would occur if elastic wall were kept fixed
Δu^{**}	= contribution to velocity fluctuation due to displacement of elastic wall
Δx^{**}	= contribution to particle displacement due to displacement of elastic wall
ε	= amplitude that measures relative strength of fluctuation in rate of heat addition due to fluctuations in effective flame position caused by flow instabilities
ζ	= pressure loss coefficient of burner
ζ_A	= pressure loss coefficient of inlet to the plenum chamber
ζ_B	= pressure loss coefficient of flame holder
λ	= number between 0 and 1
λ_0	= constant that is determined by the condition that the mean value of the solution f must vanish
ρ	= density
τ	= reaction time lag
φ	= phase angle of velocity disturbance
ω	= angular frequency
ω_n	= angular frequency that corresponds to nth mode
ω_0	= angular eigenfrequency of elastic wall

Subscripts

C	= cold air
H	= hot gas
$1, \ldots, 6$	= corresponding locations indicated in Fig. 3.

I. Introduction

THE aims of this paper are to give an overview of the thermoacoustic excitation mechanisms that may lead to strong discrete-frequency oscillations in combustion chambers of modern gas turbines and of the influence of the acoustic properties of the complete combustor system, between compressor exit and turbine inlet, on the stability and amplitudes of such oscillations. There is extensive literature on combustion instabilities in conventional combustors with so-called diffusion burners and film-cooled liners. An overview of combustion-driven oscillations has been presented by Putnam,[1] for example. The design principles of gas turbine combustors, however, have changed completely within the last decade. Extremely restrictive emission limits, particularly with respect to nitric oxides NO_x, combined with increased turbine inlet temperatures have led to the concept of lean-premixed combustion and the requirement that essentially the complete mass flow of air must

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pass through the burners to provide a sufficiently lean mixture of fuel and air in the primary zone of a combustor. For typical pressures (1500–3000 kPa) and residence times (15–30 ms) in a gas turbine combustor the gas temperature must not exceed 1700–1800 K in order to keep the emissions of NO_x below typical limits (2–25 vppm [15% O_2]). As modern alloys and turbine-blade cooling techniques already allow gas temperatures at the turbine inlet up to about 1700 K essentially all air that passes through the combustor must pass through its primary zone. For this reason there is no film cooling along the liners and no opening that would permit air to enter the combustor downstream of the primary zone of combustion. These differences between modern low-NO_x combustors and conventional diffusion type combustors are fundamentally important for all questions concerning the thermoacoustic stability properties of the complete system between the compressor exit and turbine inlet. The liner cooling systems of conventional combustors are also extremely effective sound attenuators. Resonant amplification of sound, generated as a consequence of thermoacoustic instabilities, is almost completely suppressed by liner cooling holes and mixing nozzles. In a modern low-NO_x combustor, on the other hand, the liners do not contribute to the attenuation of sound at all. Furthermore, the thermoacoustic excitation mechanisms in the case of lean-premixed combustion are very different from those of typical diffusion type burners. In summary, it can be said that the problems of thermoacoustic instability encountered with low-NO_x combustors are largely different from those of conventional gas turbine combustors but rather similar to those of rocket motors. More precisely speaking, the boundary conditions downstream of the primary zone of combustion are exactly the same for rocket motors and low-NO_x gas turbine combustors. The main differences are associated with the fuel-lean mixture and the acoustic properties of the system upstream of the burners in the latter case. It is well known that thermoacoustic instability is one of the key problems of rocket motor design. Over many decades a tremendous research effort has been made to understand the stability properties of rocket motors. Lettau[2] began investigating this problem before World War II and was followed by numerous investigators. The paper by Mitchell et al.[3] that can probably be regarded as a milestone in the more recent literature led to a rather complete understanding for the first time of the basic properties of nonlinear acoustic oscillations in rocket motors. The problem of keeping a modern gas turbine combustor stable has now become at least equally difficult as keeping a rocket motor stable, because it includes essentially all of the difficulties experienced with rocket motors, but a gas turbine should reach a life time of at least 50,000 h.

II. Forced and Self-Excited Oscillations

Prior to discussing thermoacoustic excitation mechanisms in more detail some differences between self-excited and forced oscillations should be discussed. Acoustic radiation and dissipation, due to boundary layers and shear layers, are proportional to the square of the amplitude of an acoustic oscillation, and dissipation due to shock waves is proportional to the amplitude to the third power. On the other hand, in the case of forced oscillations, the amplitude of a velocity or pressure oscillation is imposed by some mechanism that is largely independent of the acoustic oscillation. As an example, we might think of the Kelvin–Helmholtz instability of a shear layer, leading to the periodic formation of large-scale vortices in the primary zone of combustion and, as a consequence, to an oscillating rate of reaction. In this case a sound field might act as a pace maker for the formation of vortices. Phenomena of this kind have been investigated extensively in the context of wheel well noise and bomb bay buffeting (e.g., see Rossiter[4]). It has been found that the frequency of vortex rollup can lock onto the frequency of a sound field if the amplitude of the acoustic oscillation is large enough and its frequency is sufficiently close to the natural frequency of vortex shedding. From more recent investigations of forced mixing layers (e.g., see Oster and Wygnanski[5]), it may be concluded that above a certain threshold increasing forcing amplitudes do not lead to a further increase of the size of the vortices formed in the mixing layer. Hence, we can expect that fluctuations of the reaction rate that are produced by flow instabilities lead to a rate of acoustic energy addition that is proportional to the oscillation amplitude, as would be the case for an excitation due to a forced oscillating wall. As all aforementioned mechanisms of dissipation and radiation are of higher than first order in the oscillation amplitude, any type of mechanism that leads to acoustic attenuation is capable of limiting the oscillation amplitude. In the case of self-excited oscillations, however, the rate of energy addition is proportional to the square of the oscillation amplitude and, as a consequence, acoustic radiation and second-order dissipation affect the stability limits of an oscillation but cannot limit its amplitude. For more details regarding differences between forced and self-excited oscillations the reader is referred to earlier work by the present author.[6] At first it may seem that shock wave formation would be the only mechanism capable of limiting the amplitudes of self-excited oscillations, because shock wave dissipation appears to be the only third-order dissipation mechanism. Hence, it might seem that only shock waves could limit the amplitudes of self-excited oscillations. It is well known, however, that most types of thermoacoustically self-excited oscillations in combustion chambers do not contain shock waves. For the special boundary conditions considered by Mitchell et al.[3] the acoustic oscillations produced in rocket motors with concentrated combustion can be described by an equation for a Riemann invariant,

$$[f(t)+\lambda_0]\frac{\mathrm{d}f(t)}{\mathrm{d}t}+Af(t)+Bf(t-\tau)=0 \qquad (1)$$

Following the usual convention the Riemann invariants, f and g, of the downstream and upstream running sound waves are defined by

$$f=\left[\Delta u+\frac{\Delta p}{\rho a}\right]_{\text{downstream}}, \qquad g=\left[\Delta u-\frac{\Delta p}{\rho a}\right]_{\text{upstream}} \qquad (2)$$

Mitchell et al.[3] found that, depending on the oscillation period and on the values of A, B, and τ, Eq. (1) admits continuous and discontinuous solutions. Keller[6,7] discussed a somewhat more general class of self-excited oscillations that are governed by the oscillation equation

$$[f(t)+\lambda_0]\frac{\mathrm{d}f(t)}{\mathrm{d}t}+Af(t)+Bf(t-\tau)+C\frac{\mathrm{d}f(t-\tau)}{\mathrm{d}t}=0 \qquad (3)$$

So long as $C \neq 0$ and $\tau \neq 0$, Eq. (3) does not admit discontinuous solutions at all. These two examples illustrate that there is a possibility of amplitude limitation in the case of self-excited oscillations that does not require the appearance of shock waves. The effect that limits the amplitudes is associated with the nonlocal character of Eqs. (1) and (3). The terms containing the time lag τ lead to a nonlinear self-detuning of the resonator. In other words, low harmonics of an oscillation may typically be excited whereas higher harmonics may be attenuated. Nonlinear amplitude dispersion, as produced by the nonlinearity of the equation, leads to a transfer of energy from low to high harmonics. This transfer of energy grows very rapidly as the amplitude of the oscillation is increased. As a consequence, a nonlinear amplitude saturation is reached as soon as the energy transfer to higher harmonics balances the excitation of the low harmonics. Hence, this type of amplitude limitation does not require any loss of energy, as would be the case for radiation or dissipation. This type of amplitude limitation, however, only plays a significant role if nonlinear amplitude dispersion becomes strong, i.e., when the fundamental frequency of an oscillation or its amplitude is sufficiently large. Many types of self-excited oscillations in combustion chambers of gas turbines do not exhibit any signs of significant amplitude dispersion. In these cases the amplitudes are usually limited as a consequence of fuel saturation. In other words, in one part of the period the mass flow of air entering the burners is so strongly reduced that the fuel enrichment exceeds the stoichiometric limit, and complete burnout in the primary zone of combustion is no longer possible. This type of amplitude limitation was discussed by Keller et al.,[8] for example. Oscillations whose amplitudes are limited by fuel saturation typically exhibit very large amplitudes and lead to rapid destruction of engine parts.

III. Thermoacoustic Excitation Mechanisms

Mach numbers remain small throughout a gas turbine combustor, with the exception of a short domain at the end of the combustion chamber, where the Mach number increases from a value that is typically below 0.1 to 1, as the flow accelerates into the critical cross section in the first guide vanes. The situation considered is illustrated in Fig. 1. To keep the discussion simple we assume that the cross-sectional area of the combustion chamber is constant between the primary zone of combustion and the turbine inlet.

A. Sound Generated by Entropy Nonuniformities

A harmonic entropy disturbance Δs, of small amplitude, that runs into a contraction whose axial extent is short compared to the length of the entropy wave (see Fig. 2) generates an upstream running sound wave[8–10]

$$g = \frac{M_U}{1-M_U}\cdot\frac{M_D-M_U}{1+[(\gamma-1)/2]M_U M_D}\cdot\frac{a_U}{c_p}\Delta s \tag{4}$$

and the downstream running sound wave

$$f = \frac{M_D}{1-M_D}\cdot\frac{M_D-M_U}{1+[(\gamma-1)/2]M_U M_D}\cdot\left\{\frac{1+[(\gamma-1)/2]M_U^2}{1+[(\gamma-1)/2]M_D^2}\right\}^{\frac{1}{2}}\cdot\frac{a_U}{c_p}\Delta s \tag{5}$$

The turbine inlet corresponds to a contraction for which

$$M_U \ll 1 \qquad \text{and} \qquad M_D = 1 \tag{6}$$

Hence, the expression (4) can be written in the simplified form

$$g = u_H \Delta s / c_p \tag{7}$$

Accounting now for the phase changes at larger distances from the turbine inlet, an entropy disturbance

$$\Delta s = \hat{s}\exp\left(i\omega\left[t-\frac{x}{u_H}\right]\right) \tag{8}$$

that passes through the contraction at $x = L$ (see Fig. 1) generates an upstream running sound wave

$$g = u_H\frac{\hat{s}}{c_p}\exp\left(i\omega\left[t-\frac{L}{u_H}+\frac{x-L}{a_H-u_H}\right]\right) \tag{9}$$

When the upstream running sound wave arrives at the primary zone of the combustion and the burner, it generates disturbances of the mass flows of air and fuel and of the mean axial position of the flame. The pressure drop across the fuel injection nozzles, however, is usually so large that the fuel flow disturbance can be ignored. The main reason to choose a large pressure drop across the fuel injection nozzles is to prevent oscillations within the fuel supply system. The type of oscillation that appears as a consequence of an acoustic coupling between combustion chamber and fuel supply system is usually termed singing flame. For more details the reader is referred to Putnam.[1] For the present consideration we assume that the fuel flow remains undisturbed. Furthermore, it is assumed that the lengths of both sound and entropy waves are long compared to the typical dimensions of the fuel injection system and that the mean axial position of the flame is not affected by the impinging sound wave. Finally, it is assumed that the static pressure in the plenum chamber upstream of the burner (see Fig. 1) can be regarded as constant. In practical situations these assumptions often apply, as long as the oscillation frequencies are sufficiently low. Departures will be discussed later. Assuming that the primary zone of combustion is located at some distance downstream of the burner, in a domain where the cross-sectional area of the combustor has become independent of x, the following Rankine–Hugoniot jump conditions can be used to account for changes across the flame (for example, see Keller et al.[8]):

$$\frac{u_H}{u_C} = \frac{\rho_C}{\rho_H} = 1+\frac{\gamma-1}{\gamma}\cdot\frac{Q}{u_C p_C}+\mathcal{O}\left(M_C^2\right) \tag{10}$$

$$\frac{T_H}{T_C} = \left(\frac{a_H}{a_C}\right)^2 = 1+\frac{\gamma-1}{\gamma}\cdot\frac{Q}{u_C p_C}+\mathcal{O}\left(M_C^2\right) \tag{11}$$

$$\frac{p_H}{p_C} = 1-(\gamma-1)\cdot\frac{Q}{u_C p_C}\cdot M_C^2+\mathcal{O}\left(M_C^4\right) \tag{12}$$

Using Eqs. (10–12) it is easy to show that in the limit of small-amplitude disturbances and small Mach numbers the pressure and velocity disturbances are related by

$$\Delta p_H = \Delta p_C - [T_H/T_C - 1]\rho_C u_C \Delta u_C \tag{13}$$

Expressing the static pressure drop across the burner as $-\zeta\rho_C u_C^2/2$ and assuming that the pressure disturbances in the plenum chamber can be ignored, we obtain

$$\Delta p_C = -\zeta\rho_C u_C \Delta u_C \tag{14}$$

In the same limit, Eq. (10) yields

$$\Delta u_H = \Delta u_C \tag{15}$$

Combining Eqs. (10) and (13–15) we obtain

$$\Delta p_H = -\{\zeta + [T_H/T_C - 1]\}\rho_H u_H \Delta u_H \tag{16}$$

Hence, the reflection coefficient of long acoustic waves at the upstream end of the combustion chamber is

$$\frac{f}{g} = \frac{1-\{\zeta+[T_H/T_C-1]\}M_H}{1+\{\zeta+[T_H/T_C-1]\}M_H} \tag{17}$$

For the temperature disturbance downstream of the flame we obtain

$$\frac{\Delta T_H}{T_H-T_C} = -\frac{\Delta u_C}{u_C} = -\frac{T_H}{T_C}\frac{\Delta u_H}{u_H} \tag{18}$$

The corresponding entropy disturbance that is obtained using the first law of thermodynamics is given by [see Eqs. (17) and (18)]

$$u_H\cdot\frac{\Delta s}{c_p} = u_H\cdot\frac{\Delta T_H}{T_H} = -\frac{T_H-T_C}{T_C}\cdot\Delta u_H = -\frac{T_H-T_C}{2T_C}\cdot[f+g] = -\frac{T_H-T_C}{T_C}\cdot\frac{g}{1+\{\zeta+[T_H/T_C-1]\}M_H} \tag{19}$$

Combining Eqs. (7), (17), and (19) we now obtain a criterion of linear stability for a perfect resonant feed-back loop

$$\frac{[T_H/T_C-1]+|1-\{\zeta+[T_H/T_C-1]\}M_H|}{1+\{\zeta+[T_H/T_C-1]\}M_H} < 1 \tag{20}$$

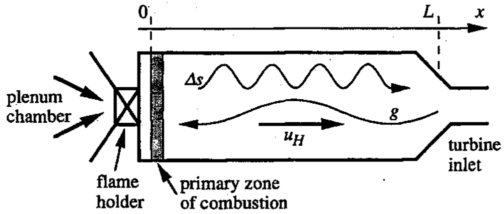

Fig. 1 Generation of sound in a combustion chamber by entropy waves interacting with the turbine inlet.

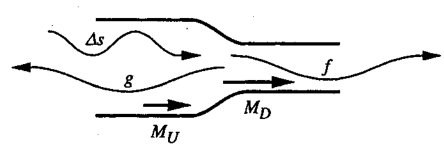

Fig. 2 Upstream and downstream running sound waves generated by an entropy disturbance interacting with a contraction in a tube.

The second term in the numerator of expression (20) can be positive (for an acoustically soft burner) or negative (for an acoustically hard burner). To include both cases the absolute value of this term has been taken. From Eqs. (8) and (9) it is evident that the oscillation frequencies, in the limit of small Mach numbers, are

$$\frac{\omega_n}{2\pi} = [2n - 1] \cdot \frac{u_H}{2L} + \mathcal{O}(M_H u_H / L) \tag{21}$$

For the lowest values of n the lengths of the sound waves are typically much longer than the combustor length L. In this case there cannot be a resonant feedback loop and, as a consequence, the stability criterion (20) reduces to

$$\frac{[T_H/T_C - 1]}{1 + \{\zeta + [T_H/T_C - 1]\}M_H} < 1 \tag{22}$$

In the case of conventional diffusion type combustors, the temperature ratio T_H/T_C may reach values as high as 5 or 6, and the loss coefficient ζ of the burner may be well below 10. For these reasons it is not surprising that sound generation due to entropy nonuniformities usually plays a major role in conventional combustors, even in the absence of resonant acoustic feedback loops.

In the case of premixed combustion, however, T_H/T_C typically does not exceed a value of about 2.5, and typical values of ζ would lie between 10 and 100. Hence, according to Eq. (22), it is quite unlikely, even in the case of very small Mach numbers, that low-frequency entropy disturbances lead to self-excited oscillations. On the other hand, because $\{\zeta + [T_H/T_C - 1]\}M_H$ is typically much larger than 1, one would expect that according to Eq. (20) entropy nonuniformities combined with resonant feedback loops would play a major role in premixed combustion. This was, indeed, true for some early types of premixing burners, where the fuel was effectively injected in only one plane. In modern premixing burners, however, the fuel is typically injected at several axial locations. As a consequence, the fluctuations in fuel enrichment caused by high-frequency acoustic waves are smeared out, and resonant feedback cycles of this type can be largely suppressed.

Nevertheless, practical experience has shown that the lowest order entropy wave modes ($n = 1$ or $n = 2$) can occasionally be observed in the case of premixed combustion. The reasons for the appearance of such ultra-low-frequency oscillations seem to be related to the proximity of the lean extinction limit. If the mass flow of air through a premixing burner is increased beyond its lean extinction limit during a certain part of an oscillation period, the corresponding temperature fluctuation may substantially exceed the values obtained from Eq. (18), because combustion is no longer complete. On the other hand, if the combustion process is still completed within the combustion chamber, in spite of periodic flame extinction, the large oscillations in the rate of heat addition may begin to play a dominant role with respect to sound generation.

Finally, it should be pointed out that in the case of sound generated by entropy disturbances an analysis of linear stability leads to a slightly optimistic judgment of the actual stability properties of a combustor. According to the nonlinear theory presented by Keller et al.[8] there is a small domain of temperature ratios on the stable side of the linear stability limit, where finite amplitude oscillations may occur in spite of the stability predicted by linear theory.

B. Sound Generated in the Primary Zone of Combustion: Self-Excited Oscillations

The term primary zone of combustion refers to the fact that some reactions are not completed at the end of this zone yet. Whereas those reactions that produce most of the heat release are completed at the end of the primary zone, slower reactions like the burnout of CO, for example, are still in progress. From the point of view of thermodynamics, however, these reactions are not particularly relevant, because they do not add any significant heat. For the subsequent consideration it is again assumed that the length of the primary zone of combustion is short compared to an acoustic wave length. Furthermore, we restrict consideration to flames that are confined to a domain of constant cross-sectional area of the combustor. On the other hand, fluctuations of the flame speed, with respect to the entering mixture of fuel and air, and the corresponding fluctuations in mean axial flame position are taken into account. Moreover, the pressure loss fluctuations that occur as a consequence of the changing flame position are included by accounting for fluctuations in the loss coefficient ζ of the burner. In this case it is more convenient to consider a frame of reference that moves with the flame, rather than the absolute frame that was considered earlier. In the relative frame the condition for mass conservation implies that

$$\rho_C u_F = \rho_H u_B \tag{23}$$

In the case of premixed combustion there is, generally, a rather substantial time lag between fuel injection and combustion. To accurately account for the history from fuel injection and mixing to combustion, we would need to express the fluctuation in the rate of heat addition in terms of convolution integrals. Being satisfied with a qualitative description, however, we may choose a discrete time lag τ to account for the delayed reaction and write

$$\frac{\Delta Q(t)}{Q} = \frac{\Delta u_C(t) - \Delta u_C(t-\tau)}{u_C} \tag{24}$$

where τ can be interpreted as the time it takes a fluid particle to move from the fuel injection system to the flame front. In the limit $\tau \to 0$ the rate of heat addition does not fluctuate, as must be the case for a fixed mass flow of fuel. Making use of the first-order relation

$$\frac{\Delta\rho_H(t)}{\rho_H} = -\frac{\Delta T_H(t)}{T_H} = \left[1 - \frac{T_C}{T_H}\right]\left[\frac{\Delta u_C(t)}{u_C} - \frac{\Delta Q(t)}{Q}\right]$$

$$= \left[1 - \frac{T_C}{T_H}\right]\frac{\Delta u_C(t-\tau)}{u_C} \tag{25}$$

that is obtained from Eqs. (10) and (11), the following relation between velocity fluctuations is obtained from Eq. (23):

$$\Delta u_F(t) = \frac{T_C}{T_H}\Delta u_B(t) + \left[1 - \frac{T_C}{T_H}\right]\Delta u_C(t-\tau) \tag{26}$$

As the mean axial flame position oscillates around the stationary position, say, $x = 0$, the time-averaged values of u_F and u_C and of u_B and u_H, respectively, must be the same. As a consequence, Eq. (13) remains to be valid, to lowest order if $\tau = 0$. Accounting for a nonzero time lag leads, with the help of Eqs. (12) and (25), to the extended form

$$\Delta p_H(t) = \Delta p_C(t) - [T_H/T_C - 1]\rho_C u_C[2\Delta u_C(t) - \Delta u_C(t-\tau)] \tag{27}$$

The Galilei transform between absolute and relative systems yields

$$\Delta u_H(t) - \Delta u_C(t) = \Delta u_B(t) - \Delta u_F(t) \tag{28}$$

The correspondingly extended form of Eq. (14) is

$$\Delta p_C(t) = -\zeta\rho_C u_C \Delta u_C(t) - \frac{1}{2}\rho_C u_C^2 \frac{d\zeta}{dX}\Delta X(t) \tag{29}$$

At this point it is convenient to restrict the consideration to harmonic fluctuations and to introduce a relation between the fluctuations in flame speed and flow speed,

$$\Delta u_F = A_0 \exp(i\varphi)\Delta u_C \tag{30}$$

Furthermore, we have

$$\Delta u_C - \Delta u_F = i\omega\Delta X \tag{31}$$

Combining Eqs. (26–31) leads to a relation between Δu_H and Δp_H,

$$\Delta p_H = -\left[\left(\zeta + [2 - \exp(-i\omega\tau)]\left[\frac{T_H}{T_C} - 1\right] + \left\{\frac{d\zeta}{dX}u_C\frac{1 - A_0\exp(i\varphi)}{2i\omega}\right\}\right)\bigg/\left(1 - [\exp(-i\omega\tau) - A_0\exp(i\varphi)]\left[\frac{T_H}{T_C} - 1\right]\right)\right]\rho_C u_C \Delta u_H \tag{32}$$

Unless ω is very small, ζ is generally much larger than all other terms in the numerator of expression (32) combined. In this case Eq. (32) can be approximated by

$$\Delta p_H = -\frac{\zeta \rho_C u_C \Delta u_H}{1 - [\exp(-i\omega\tau) - A_0 \exp(i\varphi)][T_H/T_C - 1]} \tag{33}$$

Even for very small frequencies the term within the braces in the numerator of (32) would usually not be expected to dominate, because $[1 - A_0 \exp(i\varphi)]$ becomes small, too, in this limit. According to linear stability theory, sound is generated in the primary zone of combustion if

$$\langle \Delta u_H \Delta p_H \rangle > 0 \tag{34}$$

where the averaging brackets refer to averaging with respect to time. The criterion (34) is equivalent to Rayleigh's stability criterion. According to Eq. (31) we may, therefore, conclude that sound is generated if

$$[\cos(\omega\tau) - A_0 \cos(\varphi)][(T_H/T_C) - 1] > 1 \tag{35}$$

In contrast to conventional diffusion-type combustion or two-stage premixed combustion, a time lag has no negative effect on the stability of lean-premixed combustion. The reason for the generally stabilizing effect of a combustion time lag is, of course, associated with the fact that fuel is injected and mixed close to an acoustically hard end of the combustor. However, a sufficiently large phase angle φ between the fluctuations of the mass flow through the burner and the flame speed may lead to instability. Practical experience with a variety of combustors suggests that lean-premixed combustion is generally stable, according to Eq. (35), with the exception of a certain neighborhood of the lean-blowout limit.

In this context it should be pointed out that the appearance of self-excited oscillations in a gas turbine combustor would generally lead to damages within a short time, because the oscillation amplitudes are only limited by fuel saturation, as was pointed out earlier. Hence, the amplitudes of the velocity oscillations reach the same order of magnitude as the mean flow speed in the combustor and the normalized pressure fluctuations scale with mean flow Mach number,

$$\mathcal{O}(\Delta p/p) = M \tag{36}$$

Although the Mach number of the mean flow is typically below 0.1, the amplitude of a self-excited oscillation may still become as large as 100 kPa.

C. Sound Generated in the Primary Zone of Combustion: Forced Oscillations

From the discussion a reader who is not familiar with combustion induced vibrations might get the impression that, in general, premixed combustion would not lead to significant problems of vibration. The facts are that it is a problem of great difficulty to design a gas turbine burner for premixed combustion that leads to an acceptably low level of vibration amplitudes, and that many manufacturers of gas turbines have not yet succeeded in designing lean-premixing burners that can be operated without diffusion type pilot burners. The immediate reason for these difficulties, however, is usually not related to self-excited oscillations but to flow instabilities that enforce fluctuations in the rate of heat addition. In a typical gas turbine combustor the turbulent flame speed is by more than an order of magnitude larger than the laminar flame speed. For this reason the combustion process is dominated by turbulent mixing. On the other hand, turbulent mixing exhibits strong fluctuations that can be attributed to the engulfing motion of large-scale vortices that form on shear layers, followed by rather sudden mixing within the large-scale vortices that is due to the rapid formation of secondary vortices. Broadwell and Breidenthal[11] were the first authors who discussed these fundamental aspects of mixing and chemical reaction. The process discussed leads to strong fluctuations in the rate of heat addition and, correspondingly, to the generation of sound. It seems that this type of sound generation is mainly related to the early stage of the vortex rollup process on shear layers along jets produced by flame holders. Based on arguments related to the growth rate of a shear layer on a round jet that emerges from a fully developed tube flow, for example, we might conclude that about 5% of the fresh mixture of fuel and air would be entrained during the first vortex rollup. This value is, of course, just an order of magnitude and can differ substantially from one burner to another. For the sake of argument, however, we assume that the amplitude of $\Delta Q/Q$ has a value of about 0.025, implying that 50% of the entrained mixture burns suddenly. To lowest order in the Mach number we obtain from Eqs. (10–12) and (14)

$$\frac{\Delta p_H}{\rho_H a_H} = -\zeta M_H \cdot \left[\Delta u_H - \left(1 - \frac{T_C}{T_H}\right) \cdot u_H \frac{\Delta Q}{Q}\right] \tag{37}$$

In the absence of any acoustic feedback from the combustion chamber there is no upstream running wave on the downstream side of the primary zone of combustion. Hence,

$$\Delta u_H - (\Delta p_H/\rho_H a_H) = 0 \tag{38}$$

Combining (37) and (38) we obtain

$$\frac{\Delta p_H}{p_H} = \gamma \frac{\zeta M_H}{1 + \zeta M_H}\left[1 - \frac{T_C}{T_H}\right] M_H \frac{\Delta Q}{Q} \tag{39}$$

Choosing a set of data that is typical for a modern gas turbine with premixed combustion,

$$\gamma = 1.3, \qquad \zeta = 50, \qquad M_H = 0.04$$

$$T_C/T_H = 0.4, \qquad p_H = 1600\ \text{kPa}$$

the amplitude of the pressure fluctuation is about 1.7 kPa. Vibration levels between 1 and 2 kPa are indeed typical for quiet premixed combustion. As was mentioned in Sec. II, acoustic waves that are reflected at the end of the combustor propagate upstream and may act as pacemakers for the rollup of large-scale vortices. As a consequence of the acoustic feedback from the combustor, including that produced by entropy nonuniformities and the systems upstream of the burner, the amplitudes of pressure fluctuations generated by vortex rollup and the corresponding fluctuations in the rate of heat addition may be amplified far beyond the vibration levels predicted by Eq. (39). These amplitudes often exceed the threshold that triggers the onset of self-excited lean-blowout oscillations. As soon as the amplitudes of the air mass flow fluctuations exceed a certain value periodic flame extinction will occur. To prevent situations where a forced oscillation may suddenly trigger a self-excited blowout oscillation, the burners should provide an excellent flame stabilization and fuel should be injected over an axial length that corresponds at least to the product of combustor length and Mach number upstream of the primary zone of combustion. If the latter condition is violated, resonant oscillations in the combustor can generate substantial oscillations of the equivalence ratio.

IV. Resonant Oscillations

Even in the absence of self-excited blowout oscillations, the particular acoustic properties of a combustion chamber system may lead to a significant amplification of pressure fluctuations generated as a consequence of flow instabilities in the primary zone of combustion. In contrast to conventional combustion chambers, the liners of modern lean-premixing combustors do not appreciably attenuate sound. For this reason, it is very important that the resonant frequencies of the combustor do not coincide with the natural frequencies of vortex rollup in the primary zone of combustion.

In practice there is usually no large plenum chamber upstream of the burner, as shown in Fig. 1 and, as a consequence, the static pressure upstream of the burner does not remain constant. In other words, acoustic waves are partly reflected at some upstream locations and may feed acoustic energy back into the combustor. The effects of such reflections are generally weak, unless the upstream running waves that emanate from the burner are almost completely reflected and nearly produce a node of velocity oscillations at the burner. In this case the sound attenuation that would otherwise appear due to the pressure drop across the burner could be canceled.

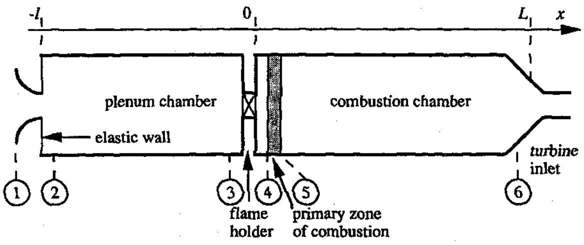

Fig. 3 Acoustic resonator consisting of a combustion chamber, a plenum chamber, and an elastic wall at the inlet to the plenum chamber.

Nearly perfect wave reflections of this type are, in general, almost impossible in a complex practical system, so long as the walls may be regarded as rigid. If the elasticity constant of an upstream wall is sufficiently small, however, an aeroelastic coupling of the system to the oscillations of this wall may lead to nearly perfect wave reflection. It is fairly simple to formulate a general limiting criterion for the elastic properties of upstream wall parts:

The acoustically generated wall displacement in the actual system with elastic walls should always be small compared to a typical acoustic particle displacement generated by the same pressure oscillation in the corresponding system with rigid walls.

Here it should be pointed out that wall parts of annular combustion chambers are typically very rigid with respect to axisymmetric excitation but often quite compliant with respect to acoustic modes that exhibit variations in the circumferential direction. Although cross modes or pure circumferential modes are more likely to appear in annular combustors than pure longitudinal modes, we restrict the present consideration to a quasi-one-dimensional system, in order to keep algebra as simple as possible. In spite of this simplification the qualitative conclusions will be more generally valid, and it is not very difficult for the experienced reader to generalize the analysis to axisymmetric or even more complicated combustion chambers and to include cross modes. To demonstrate the potential effects of elastic walls on combustion-driven oscillations, we consider the system illustrated in Fig. 3. This highly idealized situation does, of course, not reflect the complex properties and behavior of a real gas turbine combustion chamber. Nevertheless, it is useful to investigate an important property of coupled acoustic and elastic oscillations. The main limitation of a quasi-one-dimensional system is due to the fact that there are fewer choices to match resonant wave fields.

To further simplify the analysis we assume that the cross-sectional area of the combustor and the plenum chamber (see Fig. 3) are the same. Assuming that the pressure fluctuations upstream of the plenum chamber are negligibly small we can write

$$\Delta p_2 = -\zeta_A \rho_C u_C \Delta u_2^* \tag{40}$$

$$\Delta p_4 = \Delta p_3 - \zeta_B \rho_C u_C \Delta u_4 \tag{41}$$

The contribution to the mean particle displacement at position 2, which appears as a consequence of a harmonic wall displacement motion, can be expressed as

$$\Delta x_2^{**} = -\frac{\omega_0^2}{\omega_0^2 - \omega^2}\frac{\Delta p_2}{\alpha} \tag{42}$$

The expression (42) is simply the solution of the equation that governs harmonically excited oscillations and does not account for the mechanical impedance of the elastic wall. In practical situations mechanical damping is generally rather small and, except for a very small neighborhood of resonance, it can usually be ignored. In the case of a harmonic oscillation the corresponding contribution to the velocity oscillation is

$$\Delta u_2^{**} = -\frac{\omega_0^2}{\omega_0^2 - \omega^2}\frac{i\omega \Delta p_2}{\alpha} \tag{43}$$

Making use of Eqs. (40), (43), and

$$\Delta u_2 = \Delta u_2^* + \Delta u_2^{**} \tag{44}$$

we obtain

$$\Delta p_2 = -\zeta_A \rho_C u_C \left[\Delta u_2 + \frac{\omega_0^2}{\omega_0^2 - \omega^2}\frac{i\omega \Delta p_2}{\alpha}\right] \tag{45}$$

Using the Riemann invariants for the downstream and upstream running waves Eq. (45) can be written as

$$\frac{f_2}{g_2} = \left\{1 - \zeta_A M_C \cdot \left[1 - \frac{\omega_0^2}{\omega_0^2 - \omega^2}\frac{i\omega \rho_C a_C}{\alpha}\right]\right\} \Big/ \left\{1 + \zeta_A M_C \times \left[1 + \frac{\omega_0^2}{\omega_0^2 - \omega^2}\frac{i\omega \rho_C a_C}{\alpha}\right]\right\} \tag{46}$$

Hence, if

$$\alpha \gg \frac{\zeta_A M_C}{1 + \zeta_A M_C}\frac{\omega_0^2 \omega \rho_C a_C}{\left|\omega_0^2 - \omega^2\right|} \tag{47}$$

effects due to the elasticity of the wall can be ignored and Eq. (46) reduces to

$$\frac{f_2}{g_2} = \frac{1 - \zeta_A M_C}{1 + \zeta_A M_C} \tag{48}$$

On the other hand, if α is very small in the same sense, the elasticity of the wall is more important than the compressibility of the air and Eq. (46) reduces to

$$f_2/g_2 = 1 \tag{49}$$

In the latter case the elastic wall prevents sound attenuation due to jet dissipation at the inlet to the plenum chamber. If ω is sufficiently close to the angular eigenfrequency ω_0 of the elastic wall, the criterion (47) is never satisfied, provided mechanical damping remains negligibly small. To lowest order in the Mach number the Riemann invariants at the two ends of the plenum chamber are related by

$$f_3 = f_2 \exp(i\omega l/a_C), \qquad g_3 = g_2 \exp(-i\omega l/a_C) \tag{50}$$

Similarly, the Riemann invariants at the two ends of the combustor are related by

$$f_6 = f_5 \exp(i\omega L/a_H), \qquad g_6 = g_5 \exp(-i\omega L/a_H) \tag{51}$$

In the limit of long acoustic waves the reflection coefficient at the chocked throat at the end of the combustor is

$$\frac{g_6}{f_6} = -\frac{1 - [\gamma - 1]M_H/2}{1 + [\gamma - 1]M_H/2} \approx -1 \tag{52}$$

and Eq. (41) can be written in the form

$$[f_4 - g_4] = [f_3 - g_3] - \zeta_B M_C \cdot [f_4 + g_4]$$

Combining this relation with the condition for volume flow conservation across the flame holder (and thereby neglecting the mass flow of fuel added),

$$f_3 + g_3 = f_4 + g_4 \tag{53}$$

we obtain

$$f_4 = f_3 - \tfrac{1}{2}\zeta_B M_C \cdot [f_3 + g_3], \qquad g_4 = g_3 + \tfrac{1}{2}\zeta_B M_C \cdot [f_3 + g_3] \tag{54}$$

Finally, a condition needs to be found to relate Riemann invariants across the primary zone of combustion. For this purpose we may again make use of the expressions (10–12). To lowest order in the Mach number we obtain

$$\Delta p_5 = \Delta p_4, \qquad \Delta u_5 = \Delta u_4 + u_C \cdot [T_H/T_C - 1] \cdot \Delta Q/Q \tag{55}$$

In the absence of displacement fluctuations of the flame the rate of heat addition lies in the interval

$$0 \le \Delta Q^*/Q \le \Delta u_4/u_C \tag{56}$$

Table 1 Reference data

Pressure p, kPa	1500
Air temperature, K	700
Temperature of hot gas, K	1700
Amplitude ε defined by Eq. (59)	0.05
Constant of elasticity α, defined by Eq. (42), Pa*m^{-1}	10^7
Mechanical eigenfrequency of elastic wall, s^{-1}	500
Combustion parameter λ, defined by Eq. (58)	0
Pressure loss coefficient ζ_A, defined by Eq. (40)	50
Pressure loss coefficient ζ_B, defined by Eq. (41)	50
Ratio of combustor length L and fundamental resonant length L_0	1.25
Ratio of plenum chamber length l and fundamental resonant length l_0	1.25

The lower end of the interval (56) applies if the length of an acoustic wave multiplied by the Mach number of the flow in the domain of premixing of fuel and air is small compared to the axial dimension of the fuel injection system, and the upper end of the interval applies if the opposite is true. As was pointed out earlier, modern burners for premixed combustion are usually designed suitably, such that the equivalence ratio should remain nearly constant, whereas in older burner designs fuel is often injected in a single plane that is normal to the axis of symmetry of the combustor. In the latter case the lower end of the interval (56) applies. To include both limits we assume that

$$\Delta Q^*/Q = \lambda \Delta u_4/u_C \tag{57}$$

The second contribution ΔQ^{**} to the total fluctuation ΔQ is due to fluctuations in the mean (averaged over the cross-sectional area) flame position that are mostly due to vortex rollup in the primary zone of combustion. As pointed out earlier ΔQ^{**} may essentially be regarded as a forced oscillation for which the acoustic wave field acts as a pacemaker. Hence, we may write

$$\Delta Q^{**}/Q = \tfrac{1}{2}\varepsilon \exp(i\omega t) \tag{58}$$

where the amplitude ε may typically be equal to about 0.05, as was explained earlier, Eq. (37). Combining Eqs. (57) and (58) we obtain

$$\frac{\Delta Q}{Q} = \frac{\Delta Q^* + \Delta Q^{**}}{Q} = \lambda \frac{\Delta u_4}{u_C} + \frac{1}{2}\varepsilon \exp(i\omega t) \tag{59}$$

Introducing Eq. (59) in Eq. (55) and assuming that the sound speed squared is proportional to the temperature throughout the primary zone of combustion, the following relations between Riemann invariants are obtained:

$$f_5 = \frac{1}{2}\left\{\sqrt{\frac{T_H}{T_C}}[f_4 - g_4] + \left[1 + \lambda \cdot \left(\frac{T_H}{T_C} - 1\right)\right][f_4 + g_4] + \left(\frac{T_H}{T_C} - 1\right)\varepsilon u_C \exp(i\omega t)\right\}$$

$$g_5 = \frac{1}{2}\left\{-\sqrt{\frac{T_H}{T_C}}[f_4 - g_4] + \left[1 + \lambda \cdot \left(\frac{T_H}{T_C} - 1\right)\right][f_4 + g_4] + \left(\frac{T_H}{T_C} - 1\right)\varepsilon u_C \exp(i\omega t)\right\} \tag{60}$$

With the help of Eqs. (46), (50–52), (54), and (60) we can now relate all Riemann invariants to the excitation (58) and determine the resonant amplification factor.

A. Examples

To illustrate the effects of the various parameters introduced in this section we choose a list of reference data as given in Table 1 and consider the influence of different departures from the case of reference that might be regarded as typical of a gas turbine combustor. The fundamental resonant length L_0 of the combustor corresponds to half a wavelength and is computed from the eigenfrequency of the elastic wall and the sound speed of the hot gas. The fundamental resonant length l_0 of the plenum chamber corresponds to a quarter

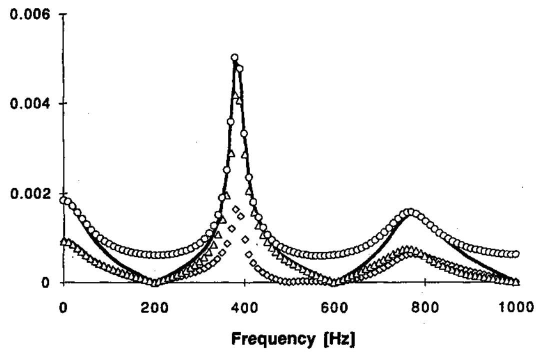

Fig. 4 Amplitudes of relative pressure fluctuations $\Delta p_2/p$, $\Delta p_3/p$, $\Delta p_4/p$, and $\Delta p_6/p$ vs frequency at the locations shown in Fig. 3 for reference data: ◊, dp2/p; △, dp3/p; ——, dp4/p; ○, dp6/p.

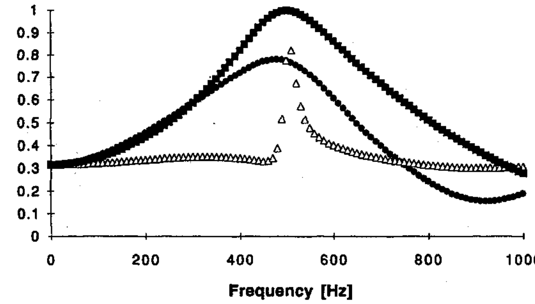

Fig. 5 Reflection coefficient f_4/g_4 vs frequency for the data sets that correspond to ●, Fig. 4; △, Fig. 6; ■, Figs. 7–9.

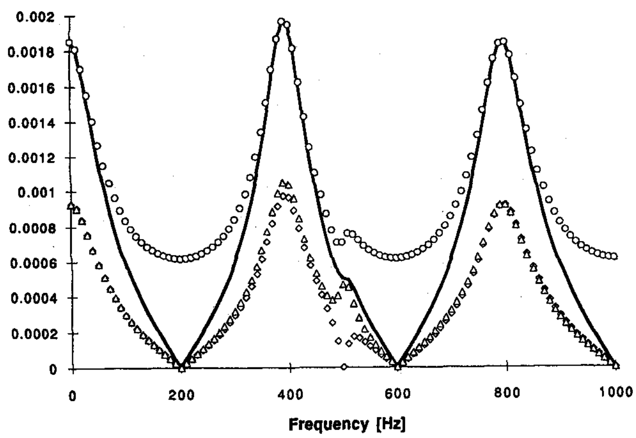

Fig. 6 Legend as for Fig. 4, except α = 10^8 Pa* m^{-1}: ◊, dp2/p; △, dp3/p; ——, dp4/p; ○, dp6/p.

of a wavelength and is obtained with the help of the sound speed of the (cold) air. The fundamental resonant lengths are $L_0 = 826$ mm and $l_0 = 265$ mm. The reference lengths are by 25% larger, $L = 1033$ mm and $l = 331$ mm. The relative amplitudes of the pressure fluctuations for the reference case at the locations 2–4 and 6, as indicated in Fig. 3, are shown in Fig. 4. The corresponding absolute value of the reflection coefficient f_4/g_4 at the burner is shown in Fig. 5. The corresponding results for a constant of elasticity that is 10 times larger than that in Table 1 are shown in Figs. 6 and 5, respectively. The comparison of Figs. 4, 5, and 7 shows the strong influence of wall elasticity if the condition (47) is not satisfied. Low constants of elasticity of wall parts may even lead to large-amplitude oscillations when the match of corresponding mechanical and acoustic eigenfrequencies is rather poor. Although walls with low constants of elasticity, in the vicinity of inlet openings, may also lead to strong sound attenuation for suitable phase relationships, there is usually such a large number of combinations of mechanical and acoustic eigenfrequencies, including, two- and three-dimensional modes,

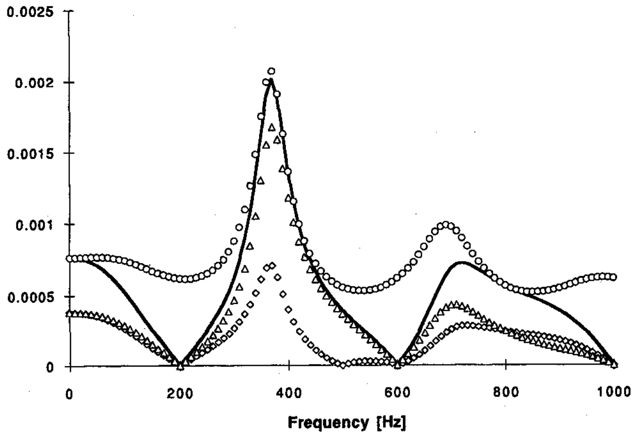

Fig. 7 Legend as for Fig. 4, except λ = 1: ◇, dp2/p; △, dp3/p; ——, dp4/p; ○, dp6/p.

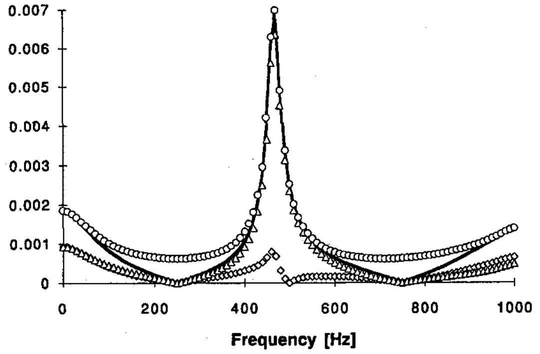

Fig. 8 Legend as for Fig. 4, except $L = L_0$: ◇, dp2/p; △, dp3/p; ——, dp4/p; ○, dp6/p.

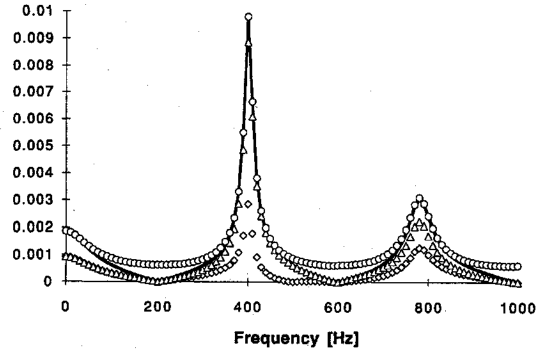

Fig. 9 Legend as for Fig. 4, except $l = l_0$: ◇, dp2/p; △, dp3/p; ——, dp4/p; ○, dp6/p.

that some of the possible combinations will generally lead to amplification. For this reason wall parts with low constants of elasticity should be avoided as far as possible. Figure 7 illustrates the effect of a long domain of fuel injection. In the case of Fig. 7 all parameters correspond to Table 1 data, except $\lambda = 1$.

Comparing Figs. 4 ($\lambda = 0$, short fuel injector) and 8 ($\lambda = 1$, long fuel injector) shows that the oscillation amplitudes are much higher for small axial dimensions of the fuel injection system. The reflection coefficient at the burner is, of course, the same as for the reference case, because the reflection coefficient depends only on the system properties upstream of location 4 (see Fig. 3). Figure 8 illustrates the effect of setting the combustor length L equal to the resonant length L_0. All other parameters correspond to the reference data. As expected, the comparison of Figs. 4 and 8 shows that the oscillation amplitudes are dramatically increased when the combustor length becomes resonant. Figure 9 illustrates the effect of setting the length l of the plenum chamber equal to its resonant length l_0. All other parameters correspond to the reference data. In the present example, it is remarkable that the effect of choosing a resonant length for the plenum chamber has an even stronger effect on the oscillation amplitudes than choosing a resonant length for the combustor. The corresponding reflection coefficient at the burner, as a function of the oscillation frequency, is also shown in Fig. 5.

V. Conclusions

In Sec. II a general overview has been given of forced and self-excited oscillations in gas turbine combustion chambers, including their key properties with respect to stability and amplitude limitation. Considerations of linear stability for a variety of excitation mechanisms discussed in Sec. III have led to the conclusion that the leading cause of combustion-driven oscillations in modern low-NO_x combustors is associated with flow instabilities that produce fluctuations in the rate of reaction. Finally, a selection of explicit results for the special case of a quasi-one-dimensional combustor has been presented in Sec. IV that should confirm some of the general conclusions in the preceding sections and demonstrate the strong influence of aeroelastic elements in a combustor.

Anticipating the problems with thermoacoustic stability of modern combustors, the power generation industry has successfully developed a variety of methods to stabilize combustors, including a very careful design with respect to the optimization of the acoustic properties of combustors and sound attenuation devices, such as Helmholtz attenuators.

Acknowledgment

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